Workshop on Advances in Variational Methods
Titles and Abstracts

June 6, 2024

Jaeyoung Byeon (Korea Advanced Institute of Science and Technology)

Title: A nonsmooth critical point theory and its application to Born-Infeld equations
Abstract: After the classical minimax argument developed by Ambrosetti and Rabinowitz, there have been many further complements of the classical theory to treat nonsmooth energy functionals. I would like to introduce our recent try to show the multiple existence of solutions to Born-Infeld equation with nonlinear sources through combining a monotonicity trick by Struwe and Jeanjean and a nonsmooth critical point theory developed by Szulkin in 1986. This talk is based on the joint work with N. Ikoma, A. Malchiodi and L. Mari.

Yuxia Guo (Tsinghua University)

Title: An elliptic problem with periodic boundary condition involving critical growth
Abstract: We study an elliptic problem involving critical growth in a strip, satisfying the periodic boundary condition. We prove that the prescribed scalar curvature problem in $\mathbb{R}^N$ has solutions which are periodic in some variables, if the scalar curvature $K(y)$ is periodic.

Louis Jeanjean (Université de Franche-Comté)

Title: On the existence of prescribed L^2 norm solutions for nonlinear Schrödinger equations on metric graphs: the mass supercritical case.
Abstract: In this talk we discuss the existence of prescribed L^2 norm solutions to nonlinear Schrödinger equations set on metric graphs. A common strategy employed to find such a solution is to search for a constrained critical point of the associated energy functional. Some geometric properties of the functional vary depending on the exponent in the nonlinear term of the equation. In the so-called mass subcritical case, the functional is bounded from below and coercive on the constraint, so one may search for a critical point as a global minimum. As such, in the last years, this case has been extensively studied. However, in the complementary case, known as the mass supercritical case, the energy functional is no longer bounded from below on the constraint and presents a lack of a priori bounds on the possible critical points. As a result, very little is yet known about this case. Through the presentation of some of the few existing results, we shall discuss the main obstacles that need to be overcome to treat this case under general assumptions. We will also present some of the tools that have already been developed for this purpose. This talk is based on joint works with J. Borthwick, P. Carrillo, X. Chang, S. Dovetta, D. Galant, X. Chang, E. Serra, N. Soave, C. Troestler.

Yanyan Li (Rutgers University)

Title: Some recent results on conformally invariant equations
Abstract: We will present some recent work on conformally invariant nonlinear elliptic equations. This includes results on Liouville type theorems, first and second derivative estimates, Study of isolated singularies of solutions, existence and nonexistence of smooth solutions.

Xi-Nan Ma (University of Science and Technology of China)

Title: Jerison-Lee identities and Semi-linear subelliptic equations on CR manifolds
Abstract: In the study of the extremal for Sobolev inequality on the Heisenberg group and the Cauchy-Riemann(CR) Yamabe problem, Jerison-Lee found a three-dimensional family of differential identities for critical exponent subelliptic equation on Heisenberg group $\mathbb{H}^n$ by using the computer in [David Jerison and John M. JAMS, 1988]. They wanted to know whether there is a theoretical framework that would predict the existence and the structure of such formulae. With the help of dimensional conservation and invariant tensors, we can answer the above question. For a class of subcritical exponent subelliptic equations on the CR manifold, several new types of differential identities are found. Then we use those identities to get the rigidity result, where rigidity means that subelliptic equations have no other solution than some constant at least when parameters are in a certain range. The rigidity result also deduces the sharp Folland-Stein inequality on closed CR manifolds. This is the joint work with Qianzhong OU and Tian WU.

Bernhard Ruf (University of Milan)

Title: On an inequality of Bliss–Moser type
Abstract: We derive a limiting inequality for the integral inequalities by Bliss. We then consider a critical version of this inequality which is of Moser type, and discuss related non-compactness properties.

Yohei Sato (Saitama University)

Title: Even symmetric ground state for the nonlinear Schrödinger system with repulsive interaction.
Abstract: In this talk, we consider the existence of even symmetric ground states for the nonlinear Schrödinger system with repulsive interaction. First, we consider the nonlinear Schrödinger system that consists of 2 equations. It is known that 2 equations' system with repulsive interaction has no ground states. However, we show that there is a possibility that there exists an even symmetric ground state. Second, we consider the nonlinear Schrödinger system that consists of 3 equations. In particular, we suppose that the system contains an attractive interaction and 2 repulsive interactions. It is known that such mixed interactions' system also has no ground states. However, we also show that there is a possibility that there exists an even ground state.

Yannick Sire (Johns Hopkins University)

Title: Eigenvalue estimates and a conjecture of Yau
Abstract: I will describe various upper and lower bounds on the spectrum of the Laplace-Beltrami on Riemannian manifolds. The upper bounds led to some important results in spectral geometry establishing a link between the so-called conformal spectrum and branched minimal immersions

into Euclidean spheres. I will then move to describe a conjecture by Yau on the first eigenvalue on minimal submanifolds of the sphere, which is known only for some examples. I will then present some recent results where we improve quantitatively the best known lower bound (in the general case) of Choi and Wang of the mid 80's. I will address some open problems and possible generalizations of our argument.

Futoshi Takahashi (Osaka University)

Title: Asymptotic behavior of radial solutions for some weighted elliptic equations on the annulus

Abstract: In this talk, we study the asymptotic behavior of radial solutions for several weighted elliptic equations with power type nonlinearities on an annulus as the nonlinear exponent $p \to +\infty$ or $p \to 1$. Asymptotic behavior of radial solutions of some weighted equations with exponential nonlinearities is also considered.

Kazunaga Tanaka (Waseda University)

Title: Normalized solutions for nonlinear Schrödinger equations with L^2 critical nonlinearity

Abstract: We study the existence of normalized solutions $u \in H^1(\mathbf{R}^N)$ for the following nonlinear Schrödinger equation:

$$\begin{cases} -\Delta u + \mu u = g(u) & \text{in } \mathbf{R}^N, \\ \frac{1}{2}\int_{\mathbf{R}^N} u^2\,dx = m, \end{cases}$$

where $N \geq 2$, $m > 0$ is a fixed mass, and $\mu > 0$ is a unknown Lagrange multiplier. We consider the case where $g(s)$ has L^2 critical growth asymptotically as $s \sim 0$ and $s \sim \pm\infty$, that is,

$$g(s) = |s|^{p-1}s + h(s), \quad p = 1 + \frac{4}{N},$$

where $h(s)$ satisfies

$$\frac{h(s)}{|s|^{p-1}s} \to 0 \quad \text{as } s \to 0 \text{ and } s \to \pm\infty.$$

We show existence and non-existence results.

This is a joint work with Silvia Cingolani, Marco Gallo and Norihisa Ikoma.

Youde Wang (AMSS, Chinese Academy of Sciences)

Title: Gradient estimates via Nash-Moser method and Liouville theorems

Abstract: We introduce how to use the Nash-Moser iteration method to obtain optimal Cheng-Yau type gradient estimates for positive solutions of a certain quasilinear degenerate elliptic equation defined on a manifold. The equation involves an unknown function, its first-order derivatives, and a power product of itself. We discuss both local and global properties of solutions, including Liouville-type properties, asymptotic behavior near singularities, and fine upper bounds on the logarithmic gradient of global solutions. The results obtained also improve or extend some existing results related to the Euclidean space scenario.

Zhi-Qiang Wang (Utah State University)

Title: Nodal solutions for a class of coupled elliptic equations
Abstract: We report results on the existence of nodal solutions for a class of coupled nonlinear elliptic equations, in particular on multiplicity results of radial nodal solutions that share prescribed nodal data in the repulsive regime. The methods are minimax constructions in the presence of invariant sets of the associated gradient or heat flows. The results show a rich structure of nodal solutions for these equations.

Shusen Yan (Central China Normal Unversity)

Title: Stable Critical Points for the Kirchhoff-Routh Type Functions
Abstract: The study of blow-up solutions will lead to the investigation of the stable critical points of the Kirchhoff-Routh type functions. In this talk, I will present some results on this aspect, with emphasis on the effects of the small holes in the domain on the existence and stability of the critical points.

Feng Zhou (East China Normal University)

Title: Eigenvalues of a Dirichlet problem involving fractional operator
Abstract: We discuss the eigenvalues of a Dirichlet problem for the fractional pseudo-relativistic Schrödinger operator in a smooth bounded domain of $\mathbb{R}^n$. The lower and upper bounds for the principle eigenvalue and the sum of the first $k-$eigenvalues are obtained by developing the Li-Yau's method and Cheng-Yang inequality. This is based on joint work with HY Chen and YH Du.

Zhitao Zhang (AMSS, Chinese Academy of Sciences)

Title: Some new results on normalized solutions of Schrodinger equations and systems
Abstract: We introduce some new results on normalized solutions, especially for normalized solutions of mass subcritical Schrodinger equations in exterior domains; normalized solutions to p-Laplacian equations with combined nonlinearities; normalized solutions to Schrodinger systems etc.

Wenming Zou (Tsinghua University)

Title: The classification of critical points of Heat kernel on tori and applications to elliptic equation and minimization problem on lattice
Abstract: In this paper, we investigate the critical points of the Heat kernel on two-dimensional flat tori. Using methods related to theta functions, we determine that the Heat kernel exhibits four and six critical points on rectangular and hexagonal tori, respectively. Furthermore, on a rhombic torus, the number of critical points of the Heat kernel depends on the geometry of torus. We have also established a connection between the Heat kernel, linear elliptic equations with singularity, and particle energy. This connection allows us to recover partial results of the Green function and provides a partial answer to Luo-Wei's conjecture (ARMA,2022) regarding the Mueller-Ho Conjecture. An intriguing finding of our study is that all three functions exhibit uniform critical points on rectangular and hexagonal tori. Jointly with C. G. Long and J.C.Wei.